Integer Programming Methods
Homework set 1

Please conform to the following instructions:

1. Make the homework in groups of 2 or 3 persons.
2. Hand in you answers as pdf file.
   (a) Use the tile "`IntPM_Homework_Set_1_<groupname>.pdf`".
   (b) At the start of your file mention your names.
   (c) The report must be clearly written, concise and complete.
3. Hand in your report by e-mail to spliet@ese.eur.nl.

Exercise 1

Consider the optimization problem

$$\begin{aligned} \min\ & \sqrt{2}x_1 - x_2 \\ 1 \leq\ & x_2 \leq \sqrt{2}x_1 \\ & x_1, x_2 \in \mathbb{N}. \end{aligned}$$

Show that this optimization has no optimal solution.

Exercise 2

Consider again the optimization problem of Exercise 1.

a. Construct feasible solutions x^k such that $\lim_{k\to\infty} \sqrt{2}x_1^k - x_2^k = 0$.

b. Prove that $\text{conv}\{x \in \mathbb{N}^2 : 1 \leq x_2 \leq \sqrt{2}x_1\}$ is not a polyhedron.

Exercise 3

Consider $P = \{x \in \mathbb{R}^3 : 0 \leq x_1, x_2, x_3 \leq 1\}$ and $Q = \{y \in \mathbb{R}^3 : y_1, y_2, y_3 \geq 0,\ y_1 + y_2 + y_3 = 1\}$. Use Fourier elimination on a system in variables x_i, y_i, z_i, to provide a system of linear inequalities describing the Minkowski sum $P + Q = \{x + y : x \in P,\ y \in Q\}$.