Integer Programming Methods
Homework set 2

Please conform to the following instructions:

1. Make the homework in groups of 2 or 3 persons.
2. Hand in you answers as pdf file.
   (a) Use the tile "`IntPM_Homework_Set_1_<groupname>.pdf`".
   (b) At the start of your file mention your names.
   (c) The report must be clearly written, concise and complete.
3. Hand in your report by e-mail to spliet@ese.eur.nl.

Exercise 1

Consider the formulation for the network design problem given in Section 2.10.2 in the book.

a. Use the block diagonal structure, where each block corresponds to an arc $a \in A$, to derive a Dantzig-Wolfe reformulation.

b. Use the block diagonal structure, where each block corresponds to a commodity $k = 1, \dots, K$, to derive a different Dantzig-Wolfe reformulation.

c. For each of these Dantzig-Wolfe reformulations, describe the pricing problem to solve the corresponding relaxation using column generation.

Exercise 2

Consider the octahedron oct_n:

$$\text{oct}_n = \left\{ \mathbf{x} \in \mathbb{R}^n : \sum_{i \in S} x_i - \sum_{i \notin S} x_i \leq 1, S \subseteq \{1, \dots, n\} \right\}.$$

a. Show that all the 2^n inequalities $\sum_{i \in S} x_i - \sum_{i \notin S} x_i \leq 1, S \subseteq \{1, \dots, n\}$ define facets of oct_n.

b. Show that oct_n has $2n$ vertices, namely the unit vectors $\mathbf{e}_i$ and their negatives $-\mathbf{e}_i$, for $i = 1, \dots, n$.

Define $P = \left\{ (\mathbf{x}, \mathbf{z}) \in \mathbb{R}^n \times \mathbb{R}^{2n} : \mathbf{x} = \sum_{i=1}^n z_i \mathbf{e}_i - \sum_{i=1}^n z_{n+i} \mathbf{e}_i, \sum_{i=1}^{2n} z_i = 1, \mathbf{z} \geq \mathbf{0} \right\}$.

c. Show that $\text{proj}_x(P) = \text{oct}_n$.

d. Show that $\dim(P) = 2n - 1$.

e. Show that P has exactly $2n$ facets, defined by the inequalities $z_i \geq 0$ for $i = 1, \dots, 2n$.